Decaying Pseudoscalars from DWF LQCD

New Horizons for Lattice Computations with Chiral Fermions
Brookhaven National Laboratory
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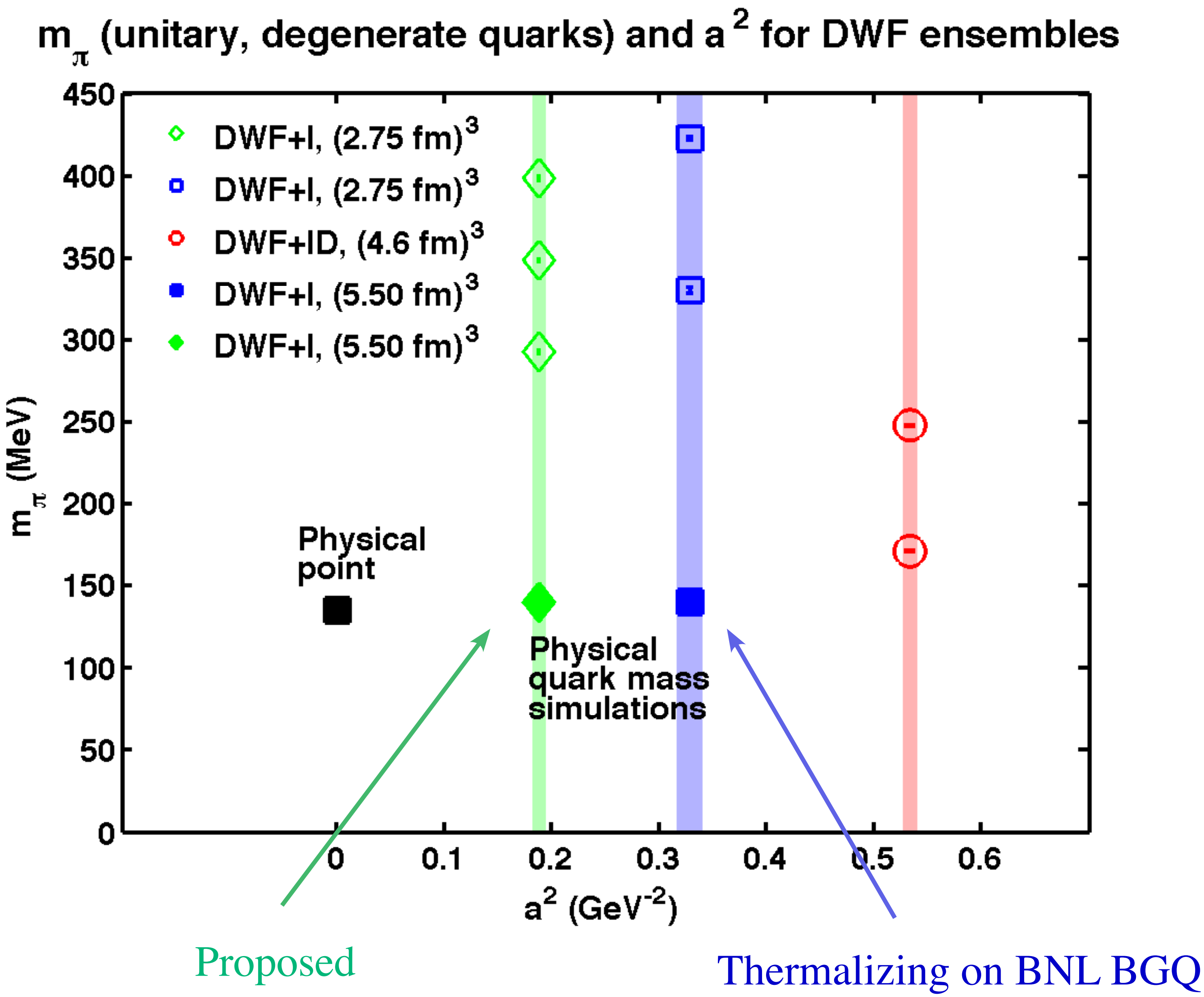
Generic Process	Examples	Experiment	LQCD calculates
$Kl2$	$K^+ \to \mu^+ \nu_\mu$ $K^+ \to e^+ \nu_e$	f_K	f_K
$Kl3$	$K^+ \to \pi^0 \, l^+ \, \nu_l$ $K^0 \to \pi^- \, l^+ \, \nu_l$	$\lvert V_{us} f^+(0) \rvert^2$	$f^+(0)$
$Kl4$	$K \to \pi \, \pi \, l \, \bar{\nu}_l$		??
$K \to \pi\pi$ (CP conserving)	$K^0 \to \pi^+ \, \pi^-$ $K^+ \to \pi^+ \, \pi^0$	$\lvert A_0 \rvert$ $\lvert A_2 \rvert$	$\lvert A_0 \rvert \; \lvert A_2 \rvert$ ($\mathrm{SM}_{\mathrm{cpc}}$ inputs)
Δm_K (CP conserving)	$K^0 \leftrightarrow \pi \, \pi \leftrightarrow \overline{K}^0$ (long distance physics) $K^0 \leftrightarrow O_{\Delta S=2} \leftrightarrow \overline{K}^0$ (short distance physics)	Δm_K	Δm_K ($\mathrm{SM}_{\mathrm{cpc}}$ inputs)
$K^0 \to \pi \; \pi$ (indirect CP violation)	$K_L \to \pi \, \pi$ $\left(K^0 \leftrightarrow \overline{K}^0\right) \to \pi \, \pi$ independent of $\pi \, \pi$ isospin	$\epsilon = \dfrac{\hat{B}_K F_K^2 \, \mathrm{SM}}{\Delta m_K}$	$\hat{B}_K$
$K^0 \to \pi \; \pi$ (direct CP violation)	$K_L \to \pi \, \pi$ depends on $\pi \, \pi$ isospin	$\mathrm{Re}(\epsilon'/\epsilon)$ $= f(A_0, A_2, \mathrm{SM})$	$A_0 \;\; A_2$ ($\mathrm{SM}_{\mathrm{cpc}}$ inputs)

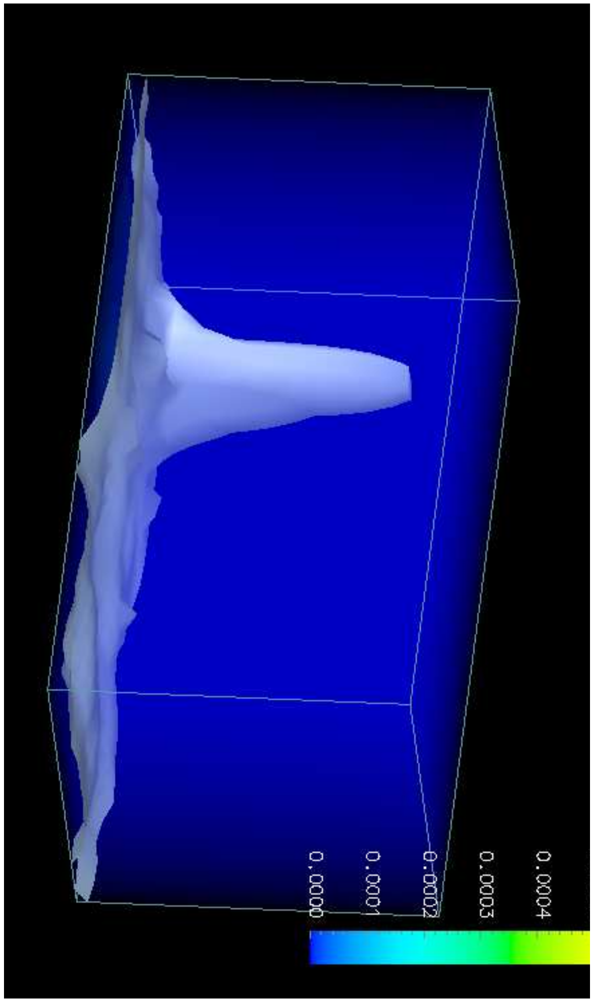
$\mathrm{SM}_{\mathrm{cpc}}$ = Standard Model CP-conserving parameters

RBC/UKQCD 2+1 flavor DWF ensembles



Improving Domain Wall Fermions

- When underlying gauge field changes topology, the DWF modes can extend farther in the fifth dimension
- This gives a non-perturbative contribution to residual chiral symmetry breaking
- Becomes problematic at strong coupling
- Add ratio of determinants of twisted Wilson fermions to suppress these gauge field dislocations
- Tune to minimize residual mass while still preserving toplogical ergodicity

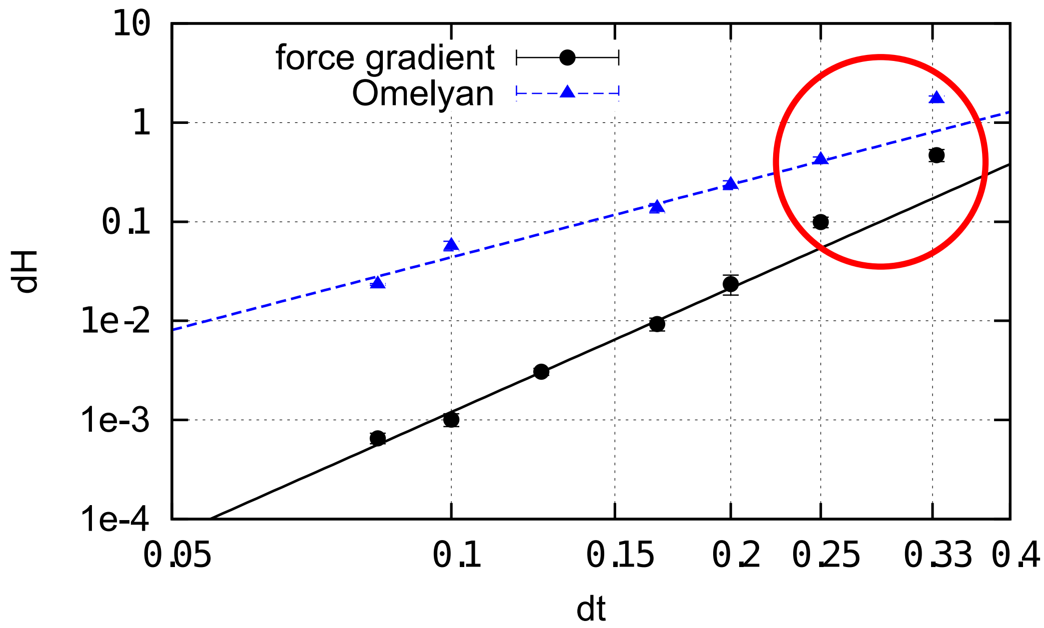


$$\frac{\det\left[D_W(-M+i\varepsilon_f\gamma^5)^\dagger D_W(-M+i\varepsilon_f\gamma^5)\right]}{\det\left[D_W(-M+i\varepsilon_b\gamma^5)^\dagger D_W(-M+i\varepsilon_b\gamma^5)\right]} = \prod_i \frac{\lambda_i^2+\varepsilon_f^2}{\lambda_i^2+\varepsilon_b^2}$$

λ_i are eigenvalues of the Hermitian Wilson operator $\gamma^5 D_W$

Force Gradient Integrator

- Proposed by Clark and Kennedy. Implemented (and simplified) in CPS by Hantao Yin
- For $16^3 \times 32 \times 16$ volumes, no speed-up compared to $O(\delta\tau^2)$ Omelyan



- For larger volumes, where δH grows with volume, force gradient may be helpful
- Tests on $48^3 \times 64 \times 16$ with 220 Mev pions using force gradient and retuning Hasenbush masses, 184 minutes/accepted configuration went down to 108 minutes/accepted configuration.

MADWF Solver

- Other chiral fermion formulations may achieve a smaller m_{res} for smaller L_s
- Mobius is one example: similar to DWF, but same m_{res} for ~ $L_s/2$
- We have many simulations at different lattice spacings to put into our global fits, so not easy to change actions. May also change topological tunneling, ...
- Idea: use Mobius fermions to accelerate the linear solver for DWF

  MADWF = Mobius Accelerated DWF
- Developed and implemented by Hantao Yin
- Gives 2× or more speed-up in quark propagator solves for current measurements.

Direct CG solve			Möbius Accelerated DWF		
operation	Op. count	time(s)	operation	Op. count	time(s)
CG solve(1e-10)	11290*32	2672	initial DWF(1e-2)	16*32	3
			DWF(1e-5)	121*32	28
			Möbius(1e-5)	4447*12	275
			DWF(1e-5)	106*32	25
			DWF(1e-5)	101*32	24
			Möbius(1e-5)	4581*12	284
			DWF(1e-5)	106*32	25
			DWF(1e-5)	102*32	24
			Möbius(1e-5)	4775*12	296
			DWF(1e-5)	106*32	25
			final DWF(1e-10)	517*32	121
total	3.61e5	2672	total	2.03e5	1138

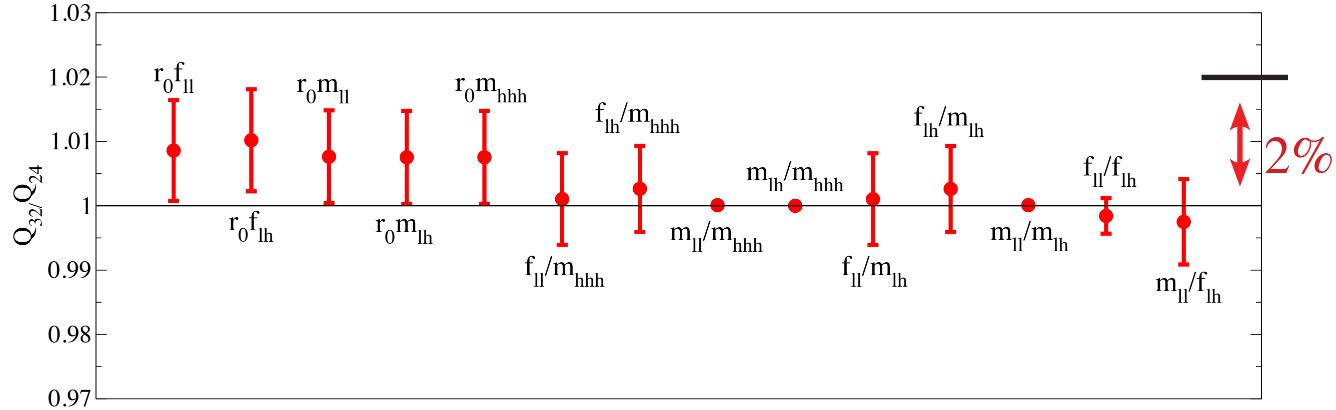
Table 1: Comparison of MADWF CG solver with a regular (zero started) CG solver. $L = 32$, $L' = 12$, with $b = 1.841556$,$c = 0.841556$. Data obtained from a 512 node partition on BG/P, both solve to 1e-10.

Scaling at unphysical light quark mass

Compare

- DWF+I: 1/a = 2.28 GeV
- DWF+I: 1/a = 1.73 GeV

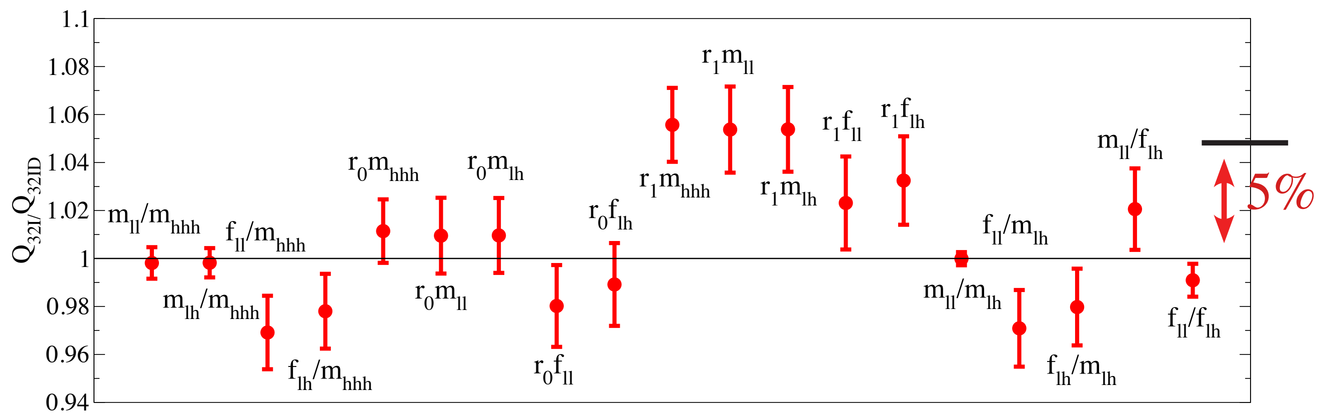
(Phys. Rev. D83 (2011) 074508)



Compare

- DWF+I: 1/a = 2.28 GeV
- DWF+ID: 1/a = 1.37 GeV

(RBC/UKQCD to appear)



See few percent scaling errors from $1/a = 1.73$ GeV $\to \infty$, with larger $O(5\%)$ errors from 1/a = 1.37 GeV

Parameters in DWF+I and DWF+ID Global Fits

- Simultaneous fit to m_π^2, m_K^2, f_π, f_K, and m_Ω
- m_π, m_K and m_Ω chosen to be quantities without $O(a^2)$ corrections
- Parameters in SU(2) chiral expansion:
  * m_π^2 and f_π: 8 parameters – 2 LO, 4 NLO, $2O(a^2)$
  * m_K^2 and f_K: 6 parameters – 2 LO, 4 NLO, $2O(a^2)$
  * m_Ω: 1 LO, 1 NLO
  * Total: 18 parameters
- Fits also determine
  * 3 lattice spacings
  * 2 ratios of light quark mass renormalization factors
  * 2 ratios of strange quark mass renormalization factors
  * m_s

Global Fits to Multiple Ensembles

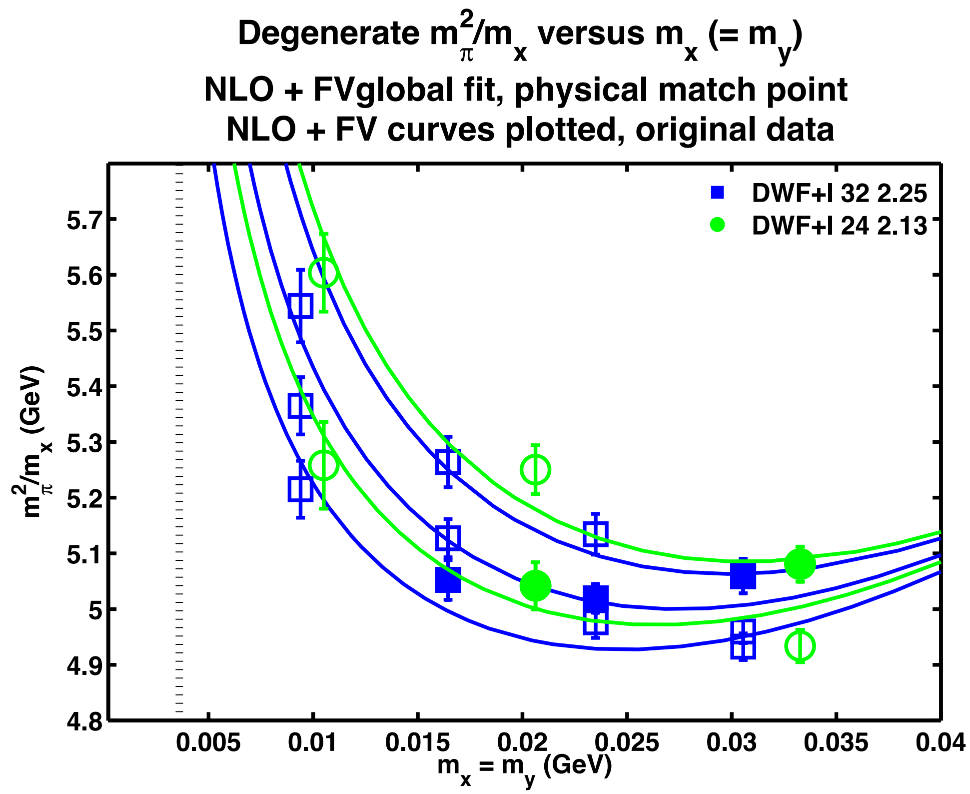
- Fit m_π^2, f_π, m_K^2, f_K and m_Ω to an expansion in powers of a^2 and m_l, including SU(2) logs where appropriate. Examples are

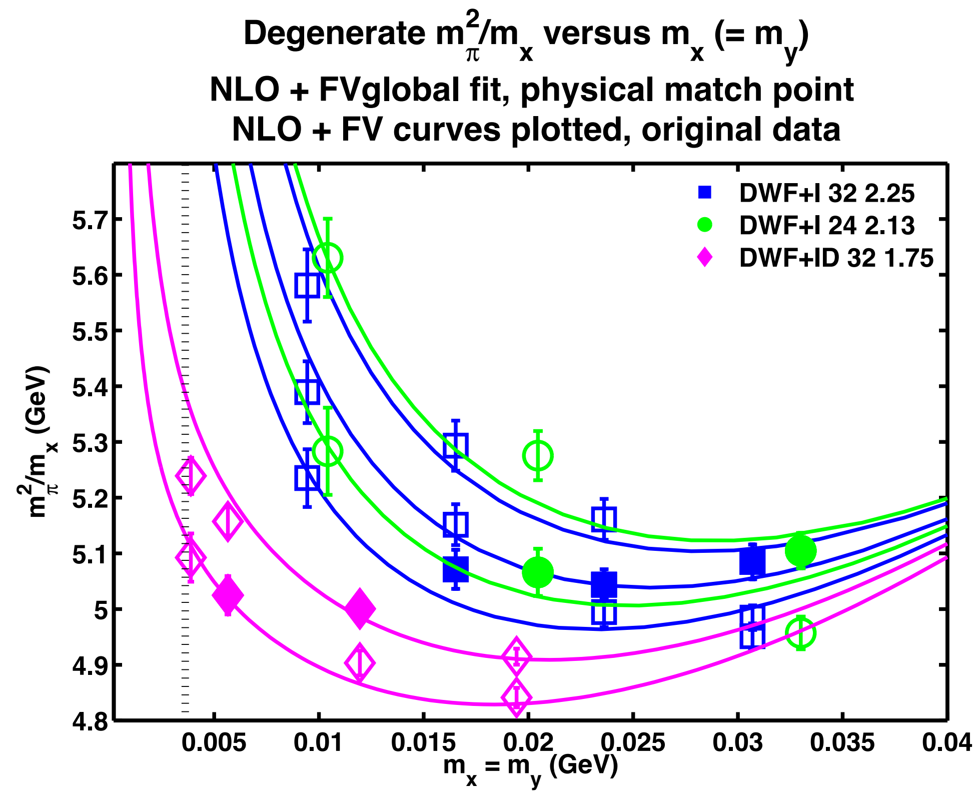
$$m_{ll}^2 = \chi_l \left[1 + \boxed{c_B a^2}\right] + \chi_l \cdot \left\{ \frac{16}{f^2}\Big((2L_8^{(2)} - L_5^{(2)}) + 2(2L_6^{(2)} - L_4^{(2)})\Big)\chi_l + \frac{1}{16\pi^2 f^2}\chi_l \log\frac{\chi_l}{\Lambda_\chi^2} \right\}$$

$$f_{ll} = f\left[1 + \boxed{c_f a^2}\right] + f \cdot \left\{ \frac{8}{f^2}(2L_4^{(2)} + L_5^{(2)})\chi_l - \frac{\chi_l}{8\pi^2 f^2}\log\frac{\chi_l}{\Lambda_\chi^2} \right\}.$$

- Note different $O(a^2)$ coefficients used for DWF+I and DWF+ID
- Fit all partially quenched data, including SU(2) ChPT finite volume corrections in fit
- Reweight data from simulation m_h to self-consistently determined m_s (Jung)
- Interpolate valence propagators to self-consistently determined m_s
- Use m_π m_K and m_Ω set scale.

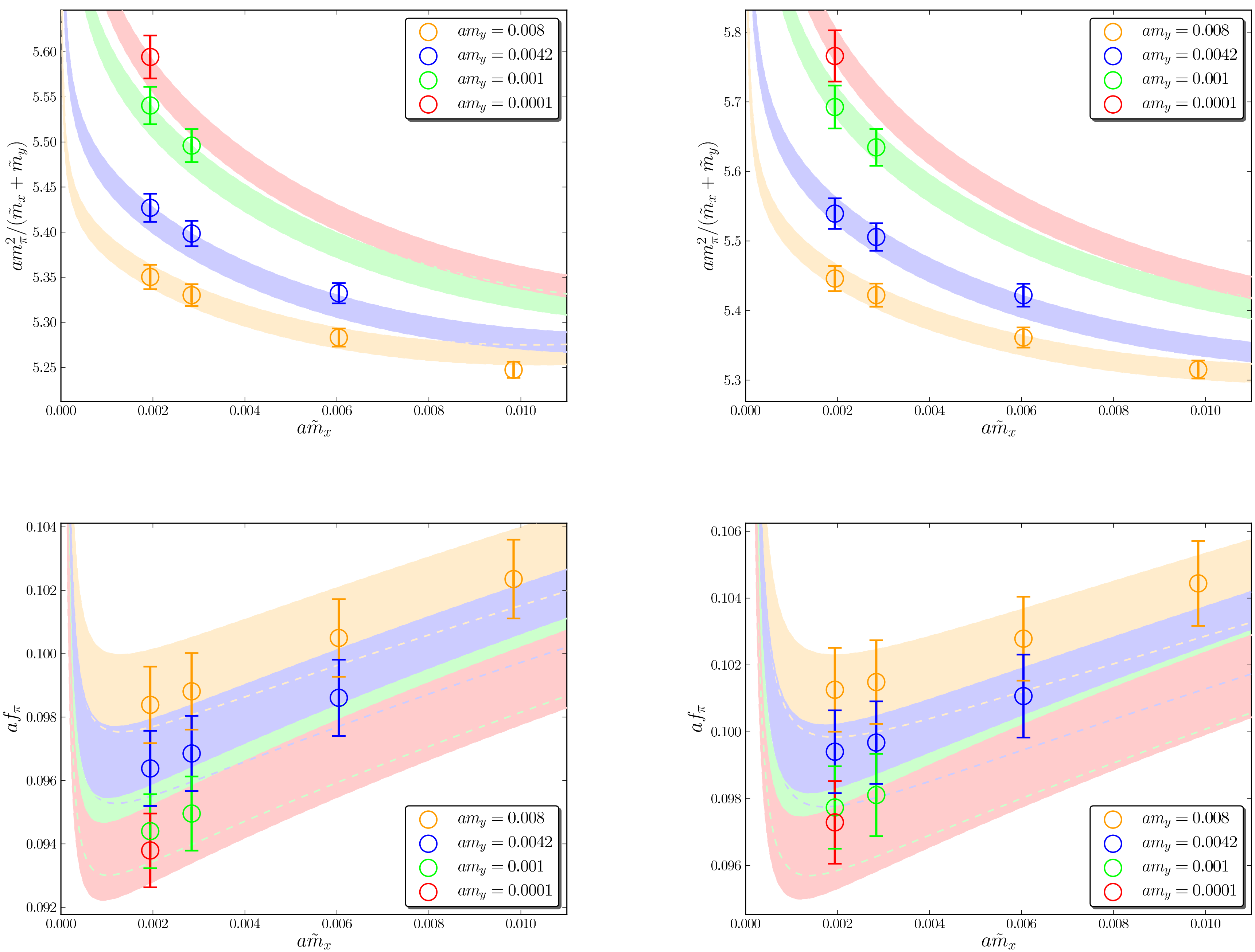
m_π^2/m_f versus m_f





- Early fits from partial DWF+ID dataset
- Data consistent with chiral logarithms

$m_\pi{}^2/m_f$ versus m_f



Some physical results

DWF+I	DWF+I and DWF+ID
$f_\pi^{\text{continuum}} = 124(2)(5)\,\text{MeV}$ $f_K^{\text{continuum}} = 149(2)(4)\,\text{MeV}$ $(f_K/f_\pi)^{\text{continuum}} = 1.204(7)(25)\,,$	$f_\pi = 127.1(2.7)(0.7)(2.5)$ MeV, $f_K = 152.4(3.0)(0.1)(1.5)$ MeV, $f_K/f_\pi = 1.1991(116)(69)(116)\,.$
$m_{ud}^{\overline{\text{MS}}}(2\,\text{GeV}) = (3.59 \pm 0.21)\,\text{MeV}$ $m_s^{\overline{\text{MS}}}(2\,\text{GeV}) = (96.2 \pm 2.7)\,\text{MeV}$ $\frac{m_s}{m_{ud}} = 26.8(0.8)_{\text{stat}}(1.1)_{\text{sys}}.$	$m_{u/d}(\overline{\text{MS}}, 3\text{ GeV}) = 3.05(8)(6)(1)(4)\,\text{MeV}.$ $m_s(\overline{\text{MS}}, 3\text{ GeV}) = 83.6(1.7)(0.7)(0.4)(1.0)\,\text{MeV}.$ $\frac{m_s}{m_{u/d}} = 27.36(39)(30)(22)(0)$
$B_K(\overline{\text{MS}}, 3\ \text{GeV}) = 0.529(5)(15)(2)(11)$	$B_K(\overline{\text{MS}}, 3\ \text{GeV}) = 0.535(8)(7)(3)(11)$ (stat, chiral, finite V, pert. theory)

Chiral extrapolation errors markedly reduced

Non-perturbative Renormalization

- Many of the quantities discussed in this talk require renormalization
- Needed to match to continuum schemes where low energy effective Hamiltonians are determined to N^nLO and renormalized at some scale μ
- Schrodinger functional and RI-MOM NPR schemes well understood
- RI-MOM is primarily used for kaons - simplicity?
- Recent improvements in RI-MOM
  - * Non-exceptional symmetric momenta - RI-SMOM
  - * Twisted b.c. to allow selection of continuous range of momenta
  - * Volume sources reduce statistical error dramatically
  - * Compute non-perturbative continuum running from fine lattices, use for coarse lattices (Rudy Arthur, Peter, Boyle, PRD 83 (2011) 114511).
  - * Implemented for $K \to \pi\pi$ (N. Garron) for RBC-UKQCD simulations on coarse lattices (1/a = 1.37 GeV).

$$\underbrace{\lim_{a_1 \to 0} \left[Z(\mu_1, a_1) Z^{-1}(\mu_0, a_1) \right]}_{\text{fine lattice}} \times \underbrace{Z(\mu_0, a_0)}_{\text{coarse lattice}} = Z(\mu_1, a_0)$$

Some $K \to \pi\,\pi$ physics

A neutral kaon beam will contain only long-lived K_L far enough from source.
Dominant decay is $K_L \to \pi\,\pi\,\pi$, small phase space gives long lifetime.
Experiments measure decay amplitudes for K_L compared to K_S (2 complex numbers).

$$\eta_{+-} = \frac{A(K_L \to \pi^+\,\pi^-)}{A(K_S \to \pi^+\,\pi^-)} \qquad \eta_{00} = \frac{A(K_L \to \pi^0\,\pi^0)}{A(K_S \to \pi^0\,\pi^0)}$$

If K_L is CP eigenstate, $\eta_{+-} = \eta_{00} = 0$.
Difference in η_{+-} and η_{00} dominantly due to difference in isospin of final state.
Consider amplitudes to decay to states of definite isospin

$$A(K^0 \to \pi\,\pi(I)) = A_I \exp(i\delta_I)$$ (I labels isospin, δ_I is $\pi\pi$ phase shift)

Parameters appearing in description of neutral kaon system

$$\eta_{+-} = \varepsilon + \frac{\varepsilon'}{1+\omega e^{i\theta'}} \approx \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - \frac{2\varepsilon'}{1-\sqrt{2}\,\omega e^{i\theta'}} \approx \varepsilon - 2\varepsilon'$$

$$\varepsilon = \tilde{\varepsilon} + i\left(\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right) \qquad \varepsilon' = \frac{\omega}{\sqrt{2}}\, e^{i\theta'}\left(\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right)$$

$$\omega = \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} = 0.045 \qquad \theta = \tan^{-1}\left[\frac{2\,\Delta M}{\Gamma_1 - \Gamma_2}\right] = 43.67 \pm 0.14^{\circ}$$

$$\theta' = \delta_2 - \delta_0 + \pi/2 = (43 \pm 6)^{\circ}$$

B_K and corrections to ε

$$\varepsilon = \boxed{\frac{e^{i\pi/4}}{\sqrt{2}\,\Delta M_K}}\left(\mathrm{Im}(M_{12}) + 2\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\mathrm{Re}(M_{12})\right)$$

This has been focus, $O(G_F^2)$ contribution from $O^{\Delta S=2}$ operator

$$B_K(\mu) \equiv \frac{\langle \bar{K}^0 | (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} | K^0 \rangle}{\frac{8}{3} f_K^2\, m_K^2}$$

Long distance physics
hep-ph/0201071 (page 58, Nierste)
Buras, Guadagnoli (PRD 78 (2008) 033005
Buras, Guadagnoli, Isidori
(PLB 688 (2010) 309

$$M_{12} = \frac{1}{2m_K}\langle K^0|H^{|\Delta S|=2}|\overline{K}^0\rangle - \boxed{\mathrm{Disp}\,\frac{i}{4m_K}\int \mathrm{d}^4x\,\langle K^0|H^{|\Delta S|=1}(x)\,H^{|\Delta S|=1}(0)|\overline{K}^0\rangle}$$

$$\Gamma_{12} = \mathrm{Abs}\,\frac{i}{2m_K}\int \mathrm{d}^4x\,\langle K^0|H^{|\Delta S|=1}(x)\,H^{|\Delta S|=1}(0)|\overline{K}^0\rangle$$

$$= \boxed{\frac{1}{2m_K}\sum_f (2\pi)^4\delta^4(p_K - p_f)\langle K^0|H^{|\Delta S|=1}|f\rangle\,\langle f|H^{|\Delta S|=1}|\overline{K}^0\rangle} \simeq \frac{1}{2m_K}A_0^*\,\overline{A}_0$$

- Norman Christ: measure these by extending Lellouch-Lüscher finite volume methods
- Jianglei Yu: numerical investigation of signal and renormalization for connected graphs

B_K

$$\varepsilon = \frac{e^{i\pi/4}}{\sqrt{2}\,\Delta M_K}\left(\mathrm{Im}(M_{12}) + 2\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\mathrm{Re}(M_{12})\right) = \kappa_\varepsilon \frac{e^{i\phi_\varepsilon}}{\sqrt{2}}\left[\frac{\mathrm{Im}\left(M_{12}^{O^{\Delta S=2}}\right)}{\Delta m_K}\right]$$

$$C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\,\pi^2\,\Delta M_K}$$

$$\lambda_t = V_{ts}^* V_{td}$$

Overall $|V_{cb}|^4$

$$\varepsilon_K = \kappa_\varepsilon C_\varepsilon \hat{B}_K \,\mathrm{Im}(\lambda_t)\{\mathrm{Re}(\lambda_c)[\eta_1 S_0(x_c) - \eta_3 S_0(x_c,x_t)] - \mathrm{Re}(\lambda_t)\,\eta_2 S_0(x_t)\}\,e^{i\pi/4}$$

0.94 ± 0.02

Buras,Guadagnoli, Isidori

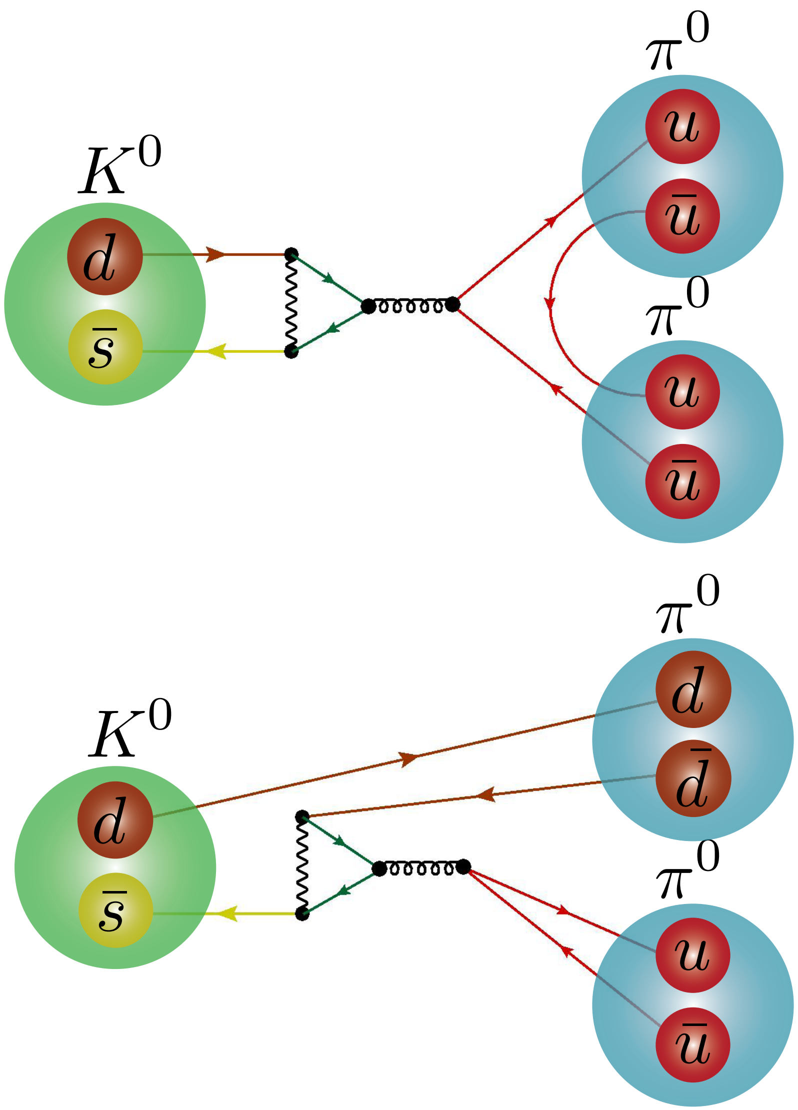
$$\eta_3^{NLO} \equiv \eta_{ct}^{NLO} \equiv 0.457 \pm 0.073$$

$$\eta_3^{NNLO} \equiv \eta_{ct}^{NNLO} \equiv 0.496 \pm 0.047$$

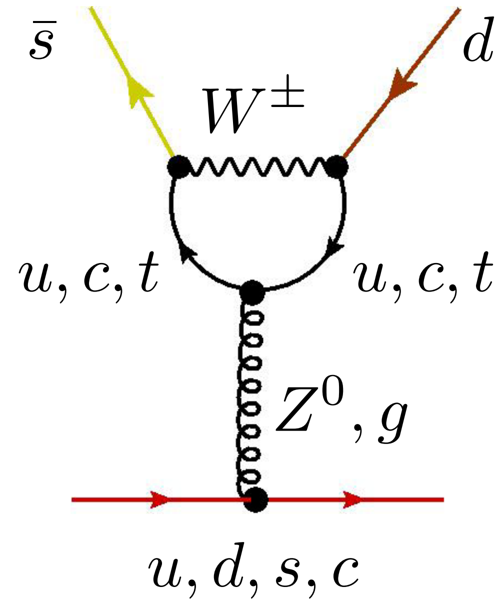
Brod and Gorbahn, PRD 82 094026 (2010)

3% overall change in ε

$K \to \pi\,\pi$ Decays via Penguins



Penguin diagram



Penguin operators

$$Q_{3,5} = (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V\mp A}$$

$$Q_{4,6} = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V\mp A}$$

$$Q_{7,9} = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V\pm A}$$

$$Q_{8,10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V\pm A}$$

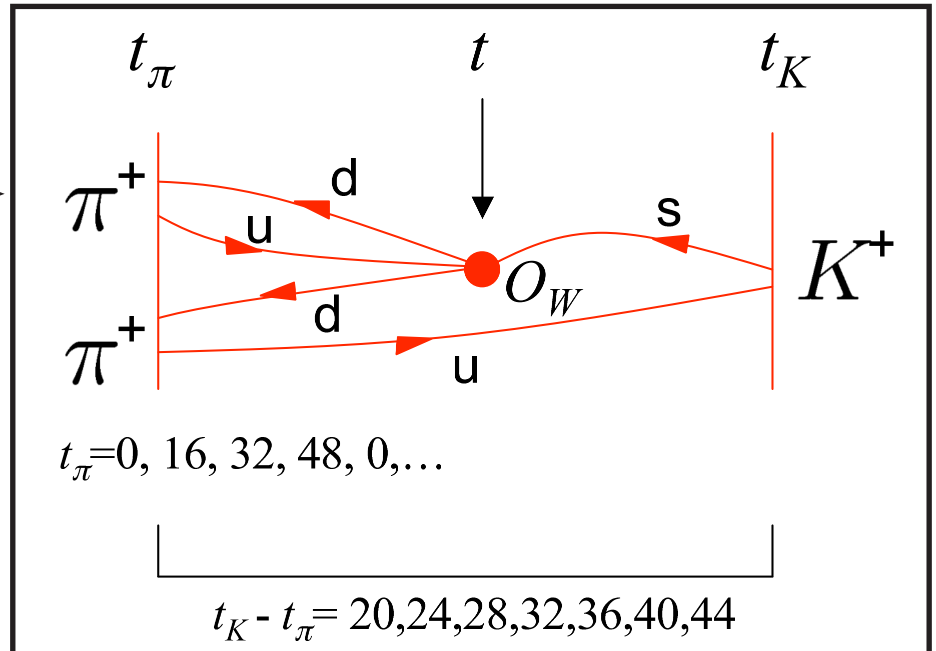
Direct calculations of $K \rightarrow \pi\pi$ $\Delta I = 3/2$ amplitudes

- RBC-UKQCD DWF+ID (Iwasaki + DSDR gauge action) ensemble

$$\mathcal{W}(M;\epsilon_f;\epsilon_b) = \frac{\det\left[D_{\mathcal{W}}(-M+i\epsilon_b\gamma^5)^\dagger D_{\mathcal{W}}(-M+i\epsilon_b\gamma^5)\right]}{\det\left[D_{\mathcal{W}}(-M+i\epsilon_f\gamma^5)^\dagger D_{\mathcal{W}}(-M+i\epsilon_f\gamma^5)\right]} = \prod_i \frac{\lambda_i^2+\epsilon_f^2}{\lambda_i^2+\epsilon_b^2}$$

- $m_\pi^{\rm dyn} = 170$ MeV, $32^3 \times 64 \times 16$ lattice volume, $(4.60\ \mathrm{fm})^3$ physical volume, $1/a = 1.37(2)$ GeV ($a = 0.146(2)$ fm), $m_{\rm res}^{\overline{MS}}(\mu = 2\ \mathrm{GeV}) = 3.7$ MeV
- $m_\pi^{\rm PQ} = 142(2)$ MeV, $m_K = 508(9)$ MeV, $\vec{p}_\pi = 199(4)$ MeV
- Physical decays have $m_\pi = 140$ MeV, $m_K = 500$ MeV, $\vec{p}_\pi = 200$ MeV

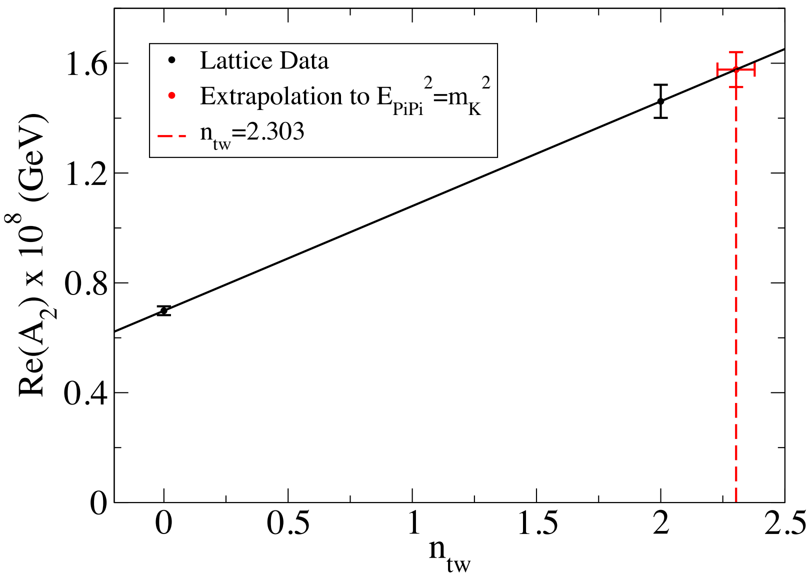
Single wall source for π's on given lattice
Multiple kaon locations, since inexpensive
Results from 62 configurations



M. Lightman and E. Goode, Lattice 2010
M. Lightman, Columbia PhD thesis, 2011
E. Goode, talk Lattice 2011

Results for $K \to \pi\pi$ $\Delta I = 3/2$ amplitudes

- Simulations also done on quenched lattices, at many kinematic points, which help to estimate errors from extrapolations to physical kinematics on unquenched lattices



Extrapolation of Re(A_2) to physical kinematics

	ReA_2	ImA_2
lattice artefacts	15%	15%
finite-volume corrections	6.2%	6.8%
partial quenching	3.5%	1.7%
renormalization	1.7%	4.7%
unphysical kinematics	3.0%	0.22%
derivative of the phase shift	0.32%	0.32%
Wilson coefficients	7.1%	8.1%
Total	18%	19%

Error estimates (M. Lightman thesis)

- N. Garron and A. Lytle have NPR results now, using 4 RI-SMOM schemes.
- Reweighting to physical light dynamical mass

$$\mathrm{Re}(A_2) = 1.397(81) \times 10^{-8}\mathrm{GeV} \xrightarrow{\text{reweighting}} 1.46(15) \times 10^{-8}\mathrm{GeV}$$

$$\mathrm{Im}(A_2) = -5.65(31) \times 10^{-13}\mathrm{GeV} \xrightarrow{\text{reweighting}} -5.79(39) \times 10^{-13}\mathrm{GeV}$$

Results for $K \to \pi\pi$ $\Delta I = 3/2$ amplitudes

- 63 configurations analyzed, in ongoing calculation.
- PRL 108 (2012) 141601

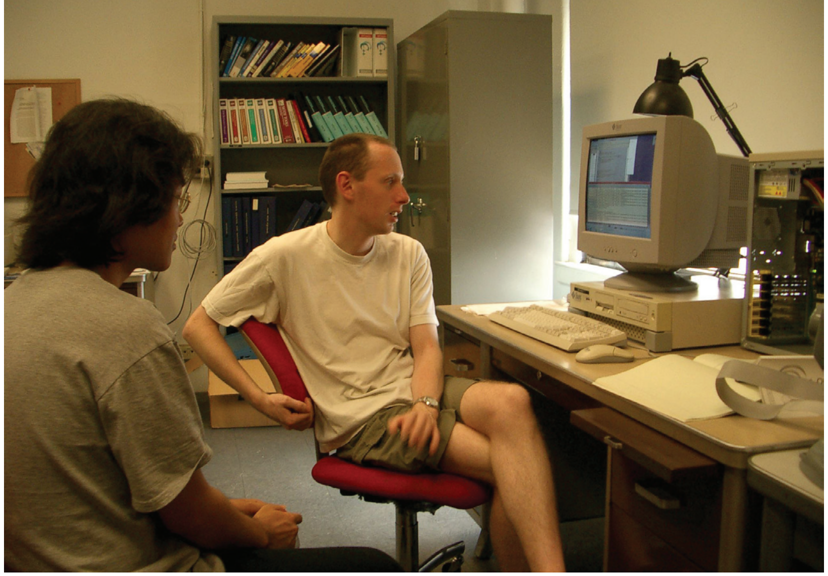
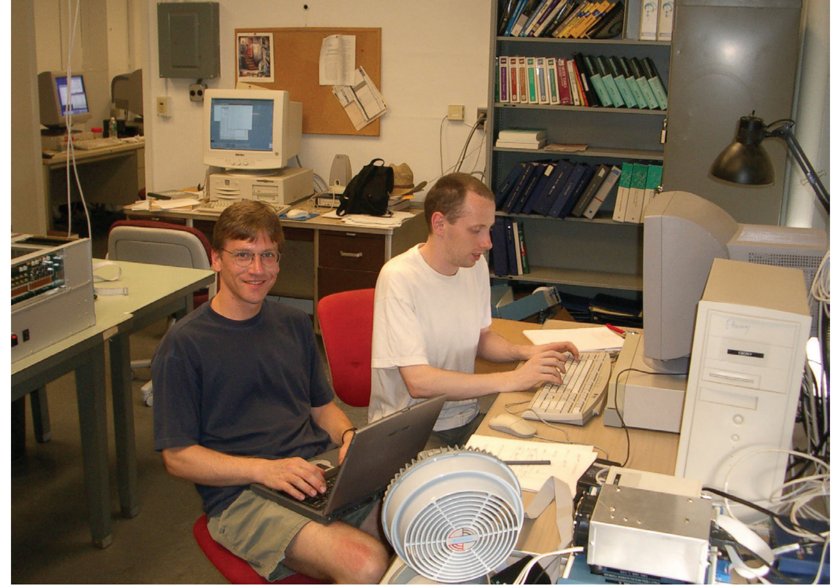
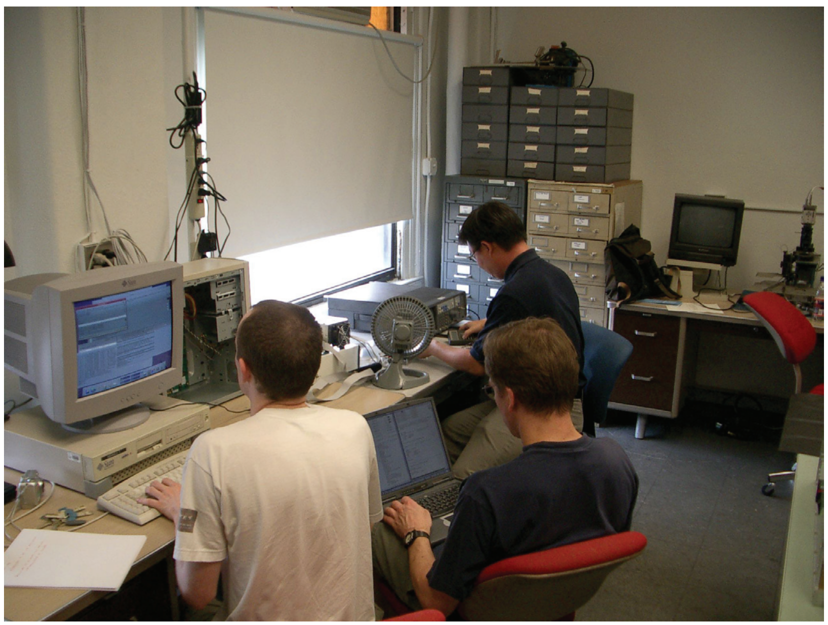
$$\mathrm{Re}\, A_2 = (1.436 \pm 0.062_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\, 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}\, A_2 = -(6.83 \pm 0.51_{\mathrm{stat}} \pm 1.30_{\mathrm{syst}})\, 10^{-13}\ \mathrm{GeV}.$$

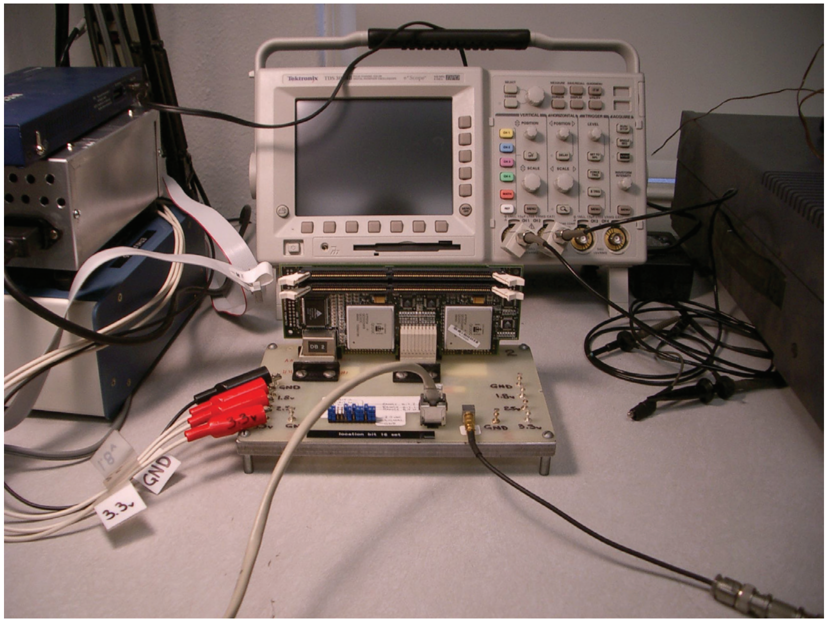
$$\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} = -4.76(37)_{\mathrm{stat}}(81)_{\mathrm{syst}} \times 10^{-5}$$

$$\mathrm{Re}(A_2) = 1.484 \times 10^{-8}\ \mathrm{GeV}\ \text{(experiment)}$$

Some observations and opinions

- With DWF (or Mobius) plus BGQ, 2+1 flavor simulations with $m_\pi = 140\ \mathrm{MeV}$ are underway
  - $48^3 \times 96 \times 32$ DWF+I with 1/a = 1.74 GeV gives $(5.5\ \mathrm{fm})^3$ box
    70 time units/BGQ-rack-month -> 500 time units/BGQ-rack-month
  - $64^3 \times 128 \times 16$ DWF+I with 1/a = 2.28 GeV gives $(5.5\ \mathrm{fm})^3$ box
    2x to 4x harder than 1/a = 1.74 GeV
  - Many hundreds of configurations with a few BGQ rack-years
  - Ideal for many physics measurements
- No chiral extrapolations!
  - Still interesting in their own right, for better determination of LEC's
  - Might need even lighter pions to know more about convergence of ChPT
  - Not an issue for real-world QCD physics
- Adding DSDR term gives viable action for finite temperature studies
- We have reached the point where 2+1 flavor QCD with full continuum symmetries, physical pions, physical kaons and large volumes can be done!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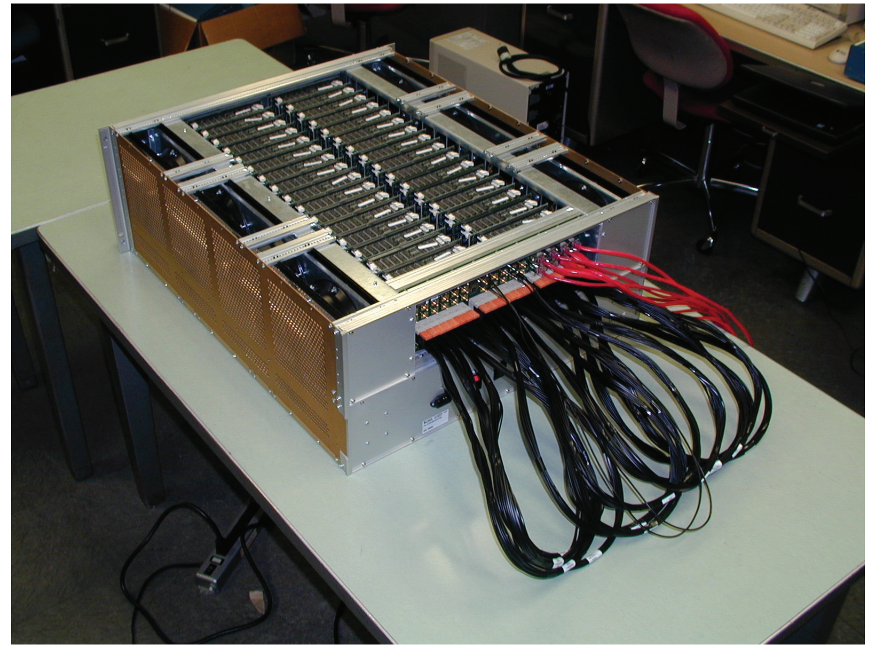
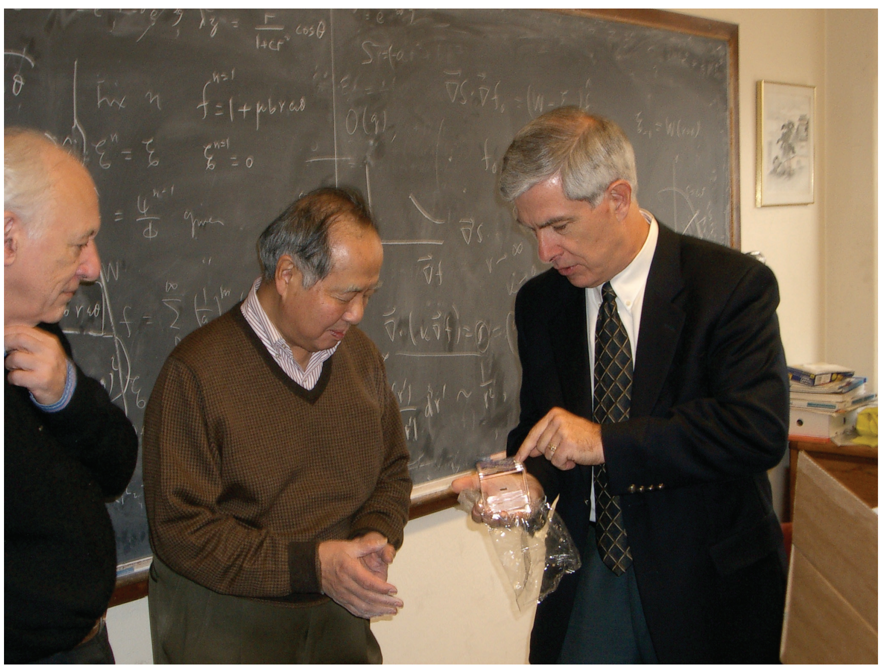
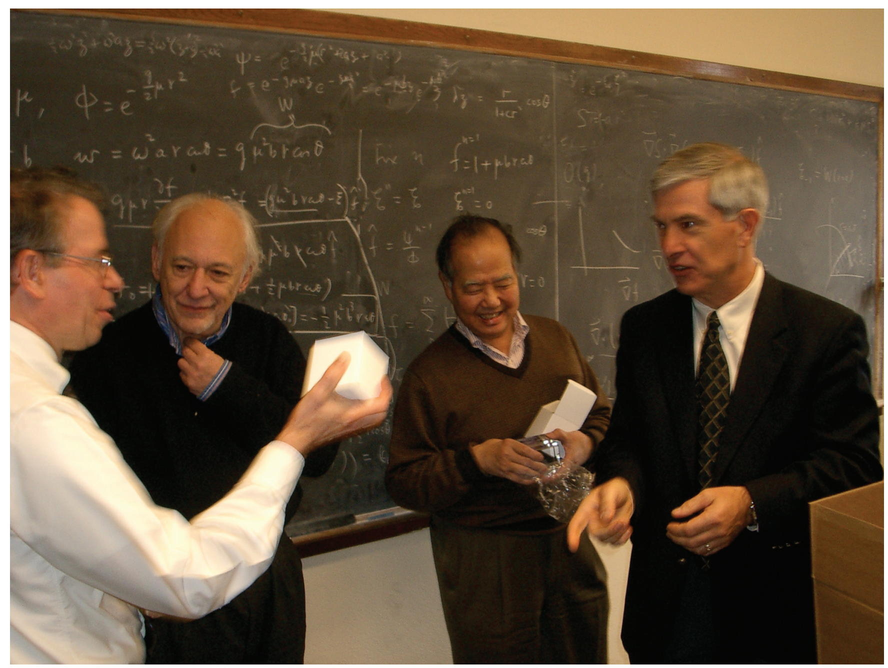
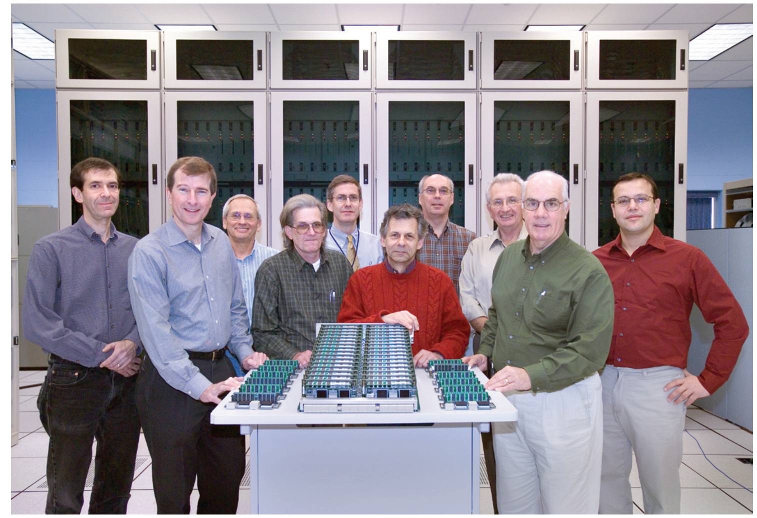




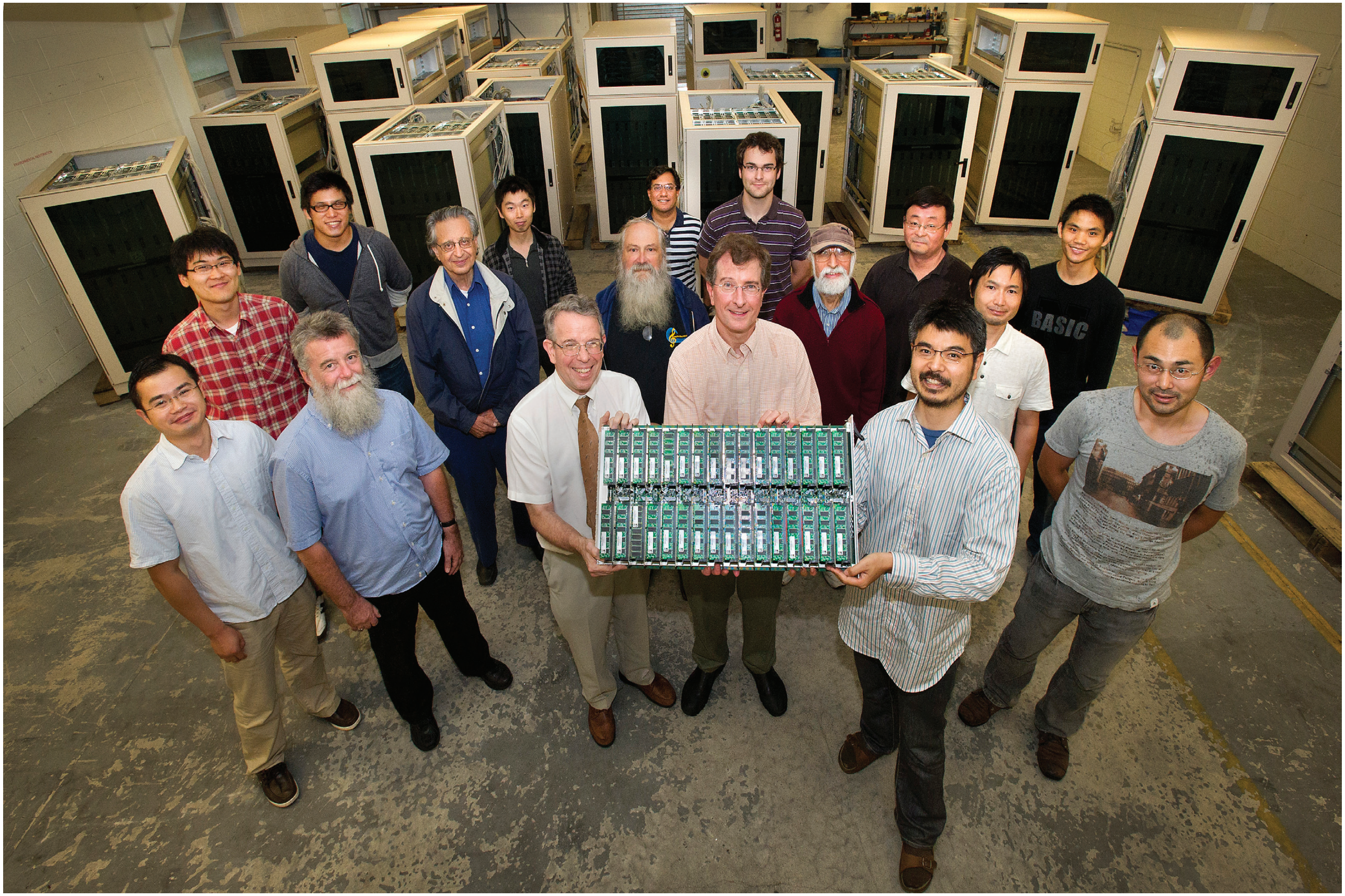





US DOE
RBRC
QCDOC at BNL
20 Teraflops

Shutdown of QCDOC, Sept. 18, 2011

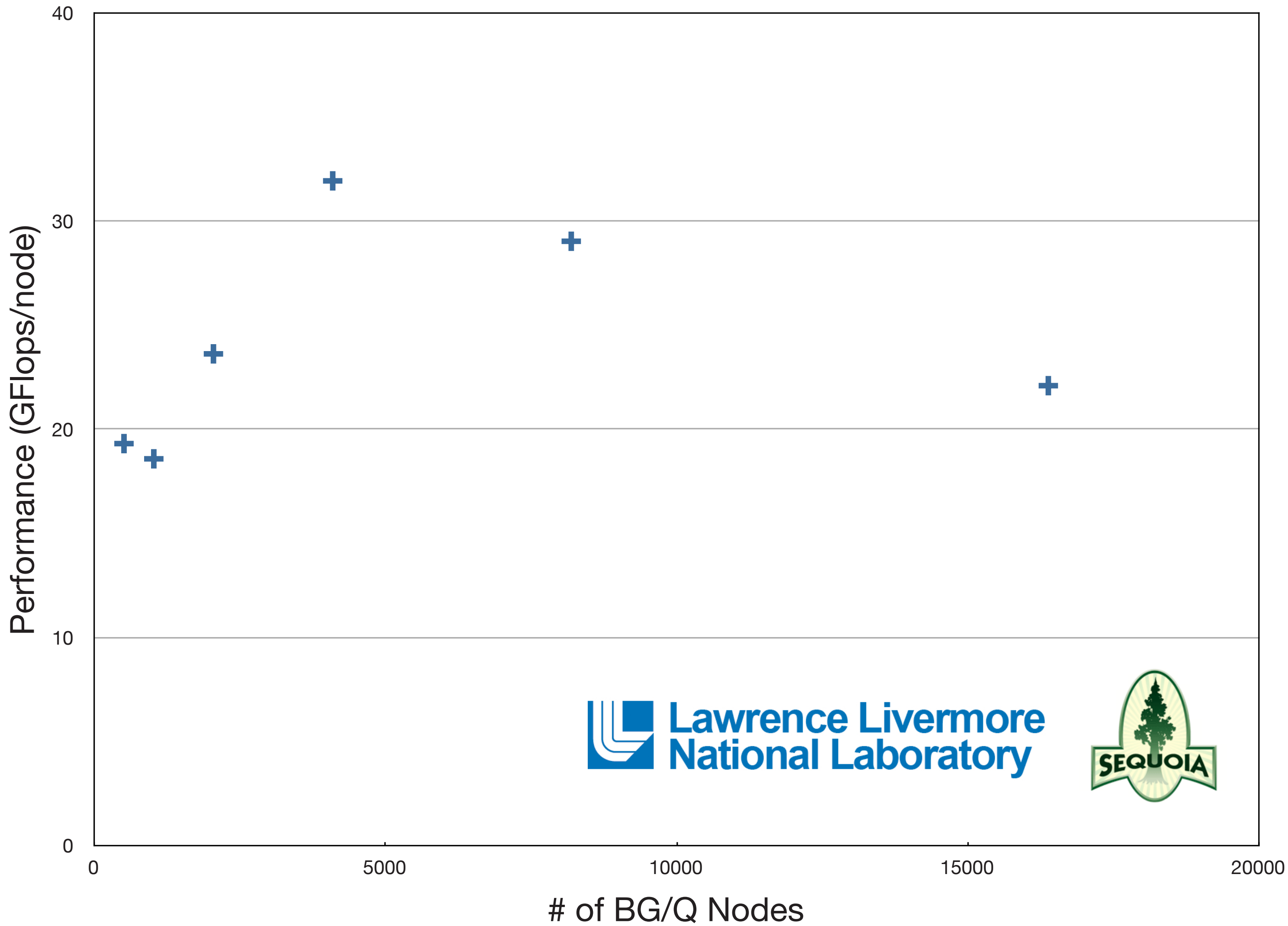
Last days of QCDSP, Nov. 2003


BASIC

BGQ at BNL

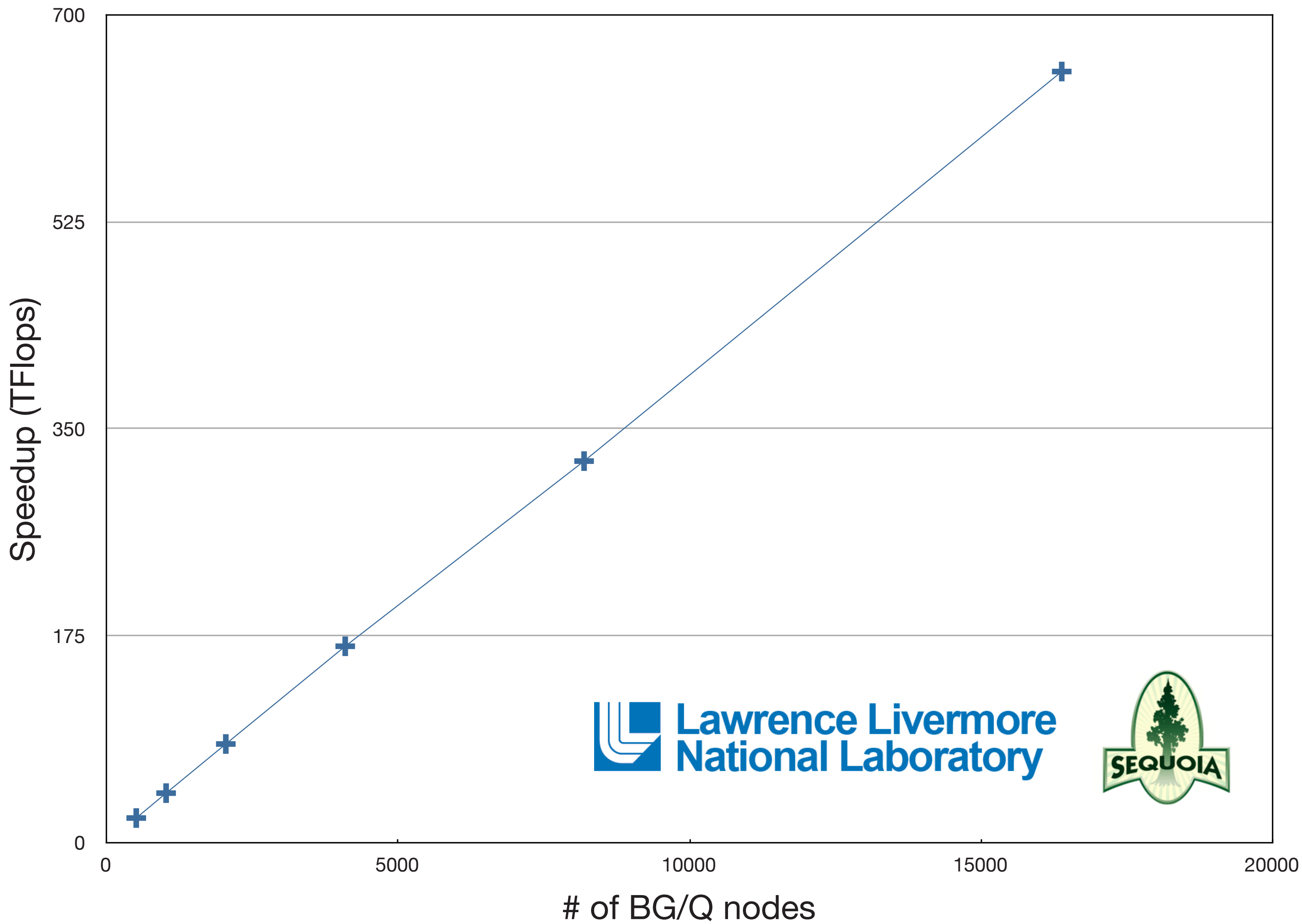
- BNL currently has 3+ racks of preproduction BGQ hardware
  - * 1 rack is owned by BNL
  - * 2 complete racks are owned by the RIKEN-BNL Research Center (RBRC)
  - * A fourth partially populated RBRC rack will be used to hold a few small BGQ partitions for code development and testing.

Strong Scaling of BAGEL DWF CG Inverter on 64^4 volume



Tests were performed with the STFC funded DiRAC facility at Edinburgh

Weak Scaling for BAGEL DWF CG Inverter



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